Project 5: Using Identities to Rewrite Expressions in Multiple Steps

Using several identities to rewrite part or all of an expression in MULTIPLE STEPS

In addition to (and often alongside) rewriting just part of an expression or equation, we may also need to apply several different identities in multiple steps in order to rewrite an expression the way that we want.

Examples:

A) Use the identities 1) $x = 1x$ to replace the second x^2 in the expression with an equivalent sub-expression and then use 2) $a - b = a + -b$ to replace the original expression with an equivalent expression that has a different form.

$$\mathbf{8x^2 - x^2}$$

First identity: Since an x appears in both the identity and in the expression, we may first want to rewrite the identity using a different letter. Let's pick z. Then $x = 1x \;\rightarrow\; z = 1z$. And to use this to replace x^2 with another equivalent sub-expression, we would do this:

$z = x^2$

$z = 1z$

$\overbrace{(\quad)}^{z} = 1\overbrace{(\quad)}^{z} \quad \rightarrow \quad \overbrace{(x^2)}^{z} = 1\overbrace{(x^2)}^{z}$

Substituting this into the **whole** expression gives us: $8x^2 - (x^2) = 8x^2 - (1x^2)$

Second identity: Now we need to use the identity $a - b = a + -b$ on this **NEW expression**. The expression $8x^2 - (1x^2)$ looks like the left half of this identity.

$a = 8x^2,\;\; b = (1x^2)$

$\overbrace{(\quad)}^{a} - \overbrace{(\quad)}^{b} = \overbrace{(\quad)}^{a} + -\overbrace{(\quad)}^{b} \quad \rightarrow \quad \overbrace{(8x^2)}^{a} - \overbrace{(1x^2)}^{b} = \overbrace{(8x^2)}^{a} + -\overbrace{(1x^2)}^{b}$

So $\boxed{\mathbf{8x^2 - x^2 = 8x^2 + -1x^2}}$

B) Use the identities 1) $a - b = a + -b$ and 2) $a(b + c) = ab + ac$ (in that order) to replace the original expression with an equivalent expression that has a different form.

$$\mathbf{3(2x - 5)}$$

First identity: This part of the expression: $2x - 5$ has the form of the left side of the identity $a - b = a + -b$.

So $a = 2x,\;\; b = 5$

$a - b = a + -b \quad \rightarrow \quad \overbrace{(\quad)}^{a} - \overbrace{(\quad)}^{b} = \overbrace{(\quad)}^{a} + -\overbrace{(\quad)}^{b} \quad \rightarrow \quad \overbrace{(2x)}^{a} - \overbrace{(5)}^{b} = \overbrace{(2x)}^{a} + -\overbrace{(5)}^{b}$

Substituting this into the **whole** expression gives us: $3(2x - 5) = 3(2x + -5)$.

Second identity: The **NEW expression** $3(2x + -5)$ has the form of the left side of the identity $a(b + c) = ab + ac$.

So $a = 3,\;\; b = 2x,\;\; c = -5$

So $a(b + c) = ab + ac \quad \rightarrow \quad \overbrace{(\quad)}^{a}\left(\overbrace{(\quad)}^{b} + \overbrace{(\quad)}^{c}\right) = \overbrace{(\quad)}^{a}\overbrace{(\quad)}^{b} + \overbrace{(\quad)}^{a}\overbrace{(\quad)}^{c}$

$\rightarrow \quad \overbrace{(3)}^{a}\left(\overbrace{(2x)}^{b} + \overbrace{(-5)}^{c}\right) = \overbrace{(3)}^{a}\overbrace{(2x)}^{b} + \overbrace{(3)}^{a}\overbrace{(-5)}^{c}$

So $\boxed{\mathbf{3(2x - 5) = (3)(2x) + (3)(-5)}}$

C) Use the identities 1) $a-b=a+-b$, 2) $a(b+c)=ab+ac$, and 3) $(b+c)a=ba+ca$ (in that order) to replace the original expression with an equivalent expression that has a different form.

$$(3x-2)(4x-1)$$

First identity: Both $(3x-2)$ and $(4x-1)$ are of the form of the left side of the identity $a-b=a+-b$. So they can each separately be replaced with equivalent sub-expressions using this identity.

For $(3x-2)$, $a=3x,\ b=2$

So $a-b=a+-b \quad \rightarrow \quad \overbrace{(\quad)}^{a}-\overbrace{(\quad)}^{b}=\overbrace{(\quad)}^{a}+-\overbrace{(\quad)}^{b} \quad \rightarrow \quad \overbrace{(3x)}^{a}-\overbrace{(2)}^{b}=\overbrace{(3x)}^{a}+-\overbrace{(2)}^{b}$

For $(4x-1)$, $a=4x,\ b=1$

So $a-b=a+-b \quad \rightarrow \quad \overbrace{(\quad)}^{a}-\overbrace{(\quad)}^{b}=\overbrace{(\quad)}^{a}+-\overbrace{(\quad)}^{b} \quad \rightarrow \quad \overbrace{(4x)}^{a}-\overbrace{(1)}^{b}=\overbrace{(4x)}^{a}+-\overbrace{(1)}^{b}$

Substituting both of these into the **whole** expression gives us: $(3x-2)(4x-1)=(3x+-2)(4x+-1)$.

Second identity: The NEW expression $(3x+-2)(4x+-1)$ has the form of the left side of the identity $a(b+c)=ab+ac$ if we define a, b and c this way:

$a=(3x+-2),\ b=4x,\ c=-1$

So $a(b+c)=ab+ac \quad \rightarrow \quad \overbrace{(\quad)}^{a}\left(\overbrace{(\quad)}^{b}+\overbrace{(\quad)}^{c}\right)=\overbrace{(\quad)}^{a}\overbrace{(\quad)}^{b}+\overbrace{(\quad)}^{a}\overbrace{(\quad)}^{c}$

$\rightarrow \quad \overbrace{(3x+-2)}^{a}\left(\overbrace{(4x)}^{b}+\overbrace{(-1)}^{c}\right)=\overbrace{(3x+-2)}^{a}\overbrace{(4x)}^{b}+\overbrace{(3x+-2)}^{a}\overbrace{(-1)}^{c}$

So $(3x+-2)(4x+-1)=(3x+-2)(4x)+(3x+-2)(-1)$

Third identity: Both $(3x+-2)(4x)$ and $(3x+-2)(-1)$ in the NEW expression have the form of the left side of the identity $(b+c)a=ba+ca$.

For $(3x+-2)(4x)$, $a=4x,\ b=3x,\ c=-2$

So $(b+c)a=ba+ca \quad \rightarrow \quad \left(\overbrace{(\quad)}^{b}+\overbrace{(\quad)}^{c}\right)\overbrace{(\quad)}^{a}=\overbrace{(\quad)}^{b}\overbrace{(\quad)}^{a}+\overbrace{(\quad)}^{c}\overbrace{(\quad)}^{a}$

$\rightarrow \quad \left(\overbrace{(3x)}^{b}+\overbrace{(-2)}^{c}\right)\overbrace{(4x)}^{a}=\overbrace{(3x)}^{b}\overbrace{(4x)}^{a}+\overbrace{(-2)}^{c}\overbrace{(4x)}^{a}$

For $(3x+-2)(-1)$, $a=-1,\ b=3x,\ c=-2$

So $(b+c)a=ba+ca \quad \rightarrow \quad \left(\overbrace{(\quad)}^{b}+\overbrace{(\quad)}^{c}\right)\overbrace{(\quad)}^{a}=\overbrace{(\quad)}^{b}\overbrace{(\quad)}^{a}+\overbrace{(\quad)}^{c}\overbrace{(\quad)}^{a}$

$\rightarrow \quad \left(\overbrace{(3x)}^{b}+\overbrace{(-2)}^{c}\right)\overbrace{(-1)}^{a}=\overbrace{(3x)}^{b}\overbrace{(-1)}^{a}+\overbrace{(-2)}^{c}\overbrace{(-1)}^{a}$

Substituting both of these into the **whole** expression gives us:

$(3x+-2)(4x)+(3x+-2)(-1)=(3x)(4x)+(-2)(4x)+(3x)(-1)+(-2)(-1)$

So $\boxed{\mathbf{(3x-2)(4x-1)=(3x)(4x)+(-2)(4x)+(3x)(-1)+(-2)(-1)}}$

D) Use the identities 1) $x=1x$ (using the whole second set of parentheses as the x for this identity), 2) $a-b=a+-b$, and 3) $a(b+c)=ab+ac$ (on the second set of parentheses only) to replace the original expression with an equivalent expression that has a different form.

$$(2x^2-5)-(x^2+2)$$

First identity: Because both our identity and our expression have x's in them, we might want to rewrite the identity using a different variable for x, in order to avoid confusion. Let's use a instead:

$x^{-n}=\frac{1}{x^n}$ (whenever $x \neq 0$) $\quad \rightarrow \quad a=1a$

This part of the expression (x^2+2) is the part that we need to treat as the a in $a=1a$.

So $a=(x^2+2)$

So $a=1a \quad \rightarrow \quad \overbrace{(\quad)}^{a}=1\overbrace{(\quad)}^{a} \rightarrow \overbrace{(x^2+2)}^{a}=1\overbrace{(x^2+2)}^{a}$

So $x^2+2=1(x^2+2)$, and substituting this into the **whole** expression gives:

$$(2x^2-5)-(x^2+2)=(2x^2-5)-1(x^2+2)$$

Second identity: The **NEW expression** $(2x^2-5)-1(x^2+2)$ has the form of the left side of the identity $a-b=a+-b$

So $a=(2x^2-5),\ \ b=1(x^2+2)$

So $a-b=a+-b \quad\rightarrow\quad \overbrace{(\ )}^{a}-\overbrace{(\ )}^{b}=\overbrace{(\ )}^{a}+-\overbrace{(\ )}^{b} \quad\rightarrow\quad \overbrace{(2x^2-5)}^{a}-\overbrace{(1(x^2+2))}^{b}=\overbrace{(2x^2-5)}^{a}+-\overbrace{(1(x^2+2))}^{b}$

So $(2x^2-5)-1(x^2+2)=(2x^2-5)+-1(x^2+2)$

Third identity: The sub-expression $-1(x^2+2)$, which is the part of the **NEW expression**, has the form of the left side of the identity $a(b+c)=ab+ac$.

So $a=-1,\ \ b=x^2,\ \ c=2$

So $a(b+c)=ab+ac \quad\rightarrow\quad \overbrace{(\ )}^{a}\left(\overbrace{(\ )}^{b}+\overbrace{(\ )}^{c}\right)=\overbrace{(\ )}^{a}\overbrace{(\ )}^{b}+\overbrace{(\ )}^{a}\overbrace{(\ )}^{c}$

$\rightarrow\quad \overbrace{(-1)}^{a}\left(\overbrace{(x^2)}^{b}+\overbrace{(2)}^{c}\right)=\overbrace{(-1)}^{a}\overbrace{(x^2)}^{b}+\overbrace{(-1)}^{a}\overbrace{(2)}^{c}$

So $-1(x^2+2)=(-1)(x^2)+(-1)(2)$ and substituting this into the **whole** expression gives:

$(2x^2-5)+-1(x^2+2)=(2x^2-5)+(-1)(x^2)+(-1)(2)$

So $\boxed{\mathbf{(2x^2-5)-(x^2+2)=(2x^2-5)+(-1)(x^2)+(-1)(2)}}$

E) Use the identity $x^{-n}=\frac{1}{x^n}$ (whenever $x\neq 0$) twice in a row to replace the original expression with an equivalent expression that has a different form.

$$\mathbf{3(2x^{-5}y)^{-4} \qquad 2x^{-5}y,\ x\neq 0}$$

First time: Because both our identity and our expression have x's in them, we might want to rewrite the identity using a different variable for x, in order to avoid confusion. Let's use a instead:

$x^{-n}=\frac{1}{x^n}$ (whenever $x\neq 0$) $\quad\rightarrow\quad a^{-n}=\frac{1}{a^n}$ (whenever $a\neq 0$)

This part of the expression $(2x^{-5}y)^{-4}$ is the part that has the form of the left side of the identity $a^{-n}=\frac{1}{a^n}$.

So $a=2x^{-5}y,\ \ n=4$

So $a^{-n}=\frac{1}{a^n} \quad\rightarrow\quad \overbrace{(\ )}^{a}{}^{-\overbrace{(\ )}^{n}}=\frac{1}{\overbrace{(\ )}^{a}{}^{\overbrace{(\ )}^{n}}} \quad\rightarrow\quad \overbrace{(2x^{-5}y)}^{a}{}^{-\overbrace{(4)}^{n}}=\frac{1}{\overbrace{(2x^{-5}y)}^{a}{}^{\overbrace{(4)}^{n}}}$ (because $2x^{-5}y\neq 0$, we have $a\neq 0$)

So $3\boxed{((2x^{-5}y)^{-4})}=3\boxed{\left(\frac{1}{(2x^{-5}y)^4}\right)}$

Second time: Again we will use the identity with a instead of x to avoid confusion. Now it is the x^{-5} part of the **NEW expression** that has the form of the left side of the identity $a^{-n}=\frac{1}{a^n}$:

So $a=x,\ \ n=5$

So $a^{-n}=\frac{1}{a^n} \quad\rightarrow\quad \overbrace{(\ )}^{a}{}^{-\overbrace{(\ )}^{n}}=\frac{1}{\overbrace{(\ )}^{a}{}^{\overbrace{(\ )}^{n}}} \quad\rightarrow\quad \overbrace{(x)}^{a}{}^{-\overbrace{(5)}^{n}}=\frac{1}{\overbrace{(x)}^{a}{}^{\overbrace{(5)}^{n}}}$ (because $x\neq 0$, we have $a\neq 0$)

So $3\left(\frac{1}{\left(2\boxed{(x^{-5})}y\right)^4}\right)=3\left(\frac{1}{\left(2\boxed{\left(\frac{1}{x^5}\right)}y\right)^4}\right)$

So $\boxed{\mathbf{3(2x^{-5}y)^{-4}=3\left(\frac{1}{\left(2\left(\frac{1}{x^5}\right)y\right)^4}\right)}}$

F) Use the identities 1) $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$ (whenever $c \neq 0$), 2) $\frac{x}{x}=1$ (whenever $x \neq 0$), 3) $(a+b)+c=a+(b+c)$, 4) $x+-x=0$, and 5) $x+0=x$ (in that order) to replace the original expression with an equivalent expression that has a different form.

$$\frac{x+y}{y}+-1 \quad (y \neq 0)$$

<u>First identity:</u> The $\frac{x+y}{y}$ part of the expression has the form of the left side of the first identity $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$.

So $a=x,\ b=y,\ c=y$ (and since $y \neq 0$, we have $c \neq 0$)

So $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c} \rightarrow \frac{\overbrace{(\ )}^{a}+\overbrace{(\ )}^{b}}{\underbrace{(\ )}_{c}}=\frac{\overbrace{(\ )}^{a}}{\underbrace{(\ )}_{c}}+\frac{\overbrace{(\ )}^{b}}{\underbrace{(\ )}_{c}} \rightarrow \frac{\overbrace{(x)}^{a}+\overbrace{(y)}^{b}}{\underbrace{(y)}_{c}}=\frac{\overbrace{(x)}^{a}}{\underbrace{(y)}_{c}}+\frac{\overbrace{(y)}^{b}}{\underbrace{(y)}_{c}}$

So substituting this back into the **whole** expression give us: $\boxed{\left(\frac{x+y}{y}\right)}+-1=\boxed{\left(\frac{x}{y}+\frac{y}{y}\right)}+-1$

<u>Second identity:</u> The $\frac{y}{y}$ part of the <u>**NEW expression**</u> has the form of the left side of the second identity $\frac{x}{x}=1$.

So $x=y$ (and since $y \neq 0$, we have $x \neq 0$)

So $\frac{x}{x}=1 \rightarrow \frac{\overbrace{(\ )}^{x}}{\underbrace{(\ )}_{x}}=1 \rightarrow \frac{\overbrace{(y)}^{x}}{\underbrace{(y)}_{x}}=1$

So substituting this into the **whole** expression gives: $\left(\frac{x}{y}+\boxed{\left(\frac{y}{y}\right)}\right)+-1=\left(\frac{x}{y}+\boxed{(1)}\right)+-1$ which is just $\left(\frac{x}{y}+1\right)+-1$

<u>Third identity:</u> The whole NEW expression now has the form of the left side of the identity $(a+b)+c=a+(b+c)$.

So $a=\frac{x}{y},\ \ b=1,\ \ c=-1$

So $(a+b)+c=a+(b+c) \rightarrow \left(\overbrace{(\ )}^{a}+\overbrace{(\ )}^{b}\right)+\overbrace{(\ )}^{c}=\overbrace{(\ )}^{a}+\left(\overbrace{(\ )}^{b}+\overbrace{(\ )}^{c}\right)$

$\rightarrow \left(\overbrace{\left(\frac{x}{y}\right)}^{a}+\overbrace{(1)}^{b}\right)+\overbrace{(-1)}^{c}=\overbrace{\left(\frac{x}{y}\right)}^{a}+(\overbrace{(1)}^{b}+\overbrace{(-1)}^{c})$

So this is just: $\left(\frac{x}{y}+1\right)+-1=\frac{x}{y}+(1+-1)$

<u>Fourth identity:</u> This part of the <u>**NEW expression**</u> $(1+-1)$ now has the form of the left side of the identity $x+-x=0$

So $x=1$

So $x+-x=0 \rightarrow \overbrace{(\ )}^{x}+-\overbrace{(\ )}^{x}=0 \rightarrow \overbrace{(1)}^{x}+-\overbrace{(1)}^{x}=0$

So substituting this into the **whole** expression gives us: $\frac{x}{y}+\boxed{(1+-1)}=\frac{x}{y}+\boxed{(0)}$ which is just $\frac{x}{y}+0$

<u>Fifth identity:</u> The whole expression now has the form of the left side of the identity $x+0=x$. But since both the identity and the expression have x's in them, we may want to rewrite the identity using a different variable. Let's use c instead. Then $x+0=x \rightarrow c+0=c$

So $c=\frac{x}{y}$

So $c+0=c \rightarrow \overbrace{(\ )}^{c}+0=\overbrace{(\ )}^{c} \rightarrow \overbrace{\left(\frac{x}{y}\right)}^{c}+0=\overbrace{\left(\frac{x}{y}\right)}^{c}$

So we can rewrite $\frac{x}{y}+0=\frac{x}{y}$

And therefore $\boxed{\frac{x+y}{y}+-1=\frac{x}{y}}$ **(as long as $y \neq 0$)**

<u>Now you try!</u> For each of the following problems, use the given identities to replace the expression with another equivalent expression with a different form. **Use the examples above as a model, writing out all of the same steps.** **<u>Be sure to apply the identity from each step to the NEW expression that is the END result of the previous step</u>**<u>—do NOT simply apply the identity in steps 2 or later to the *original* expression.</u>

1. Use the identities 1) $a(b+c) = ab + ac$ and 2) $(b+c)a = ba + ca$ (in that order) to replace the original expression with an equivalent expression that has a different form.

$$(2x+1)(x+5)$$

2. Use the identities 1) $x = 1x$ (using the whole second set of parentheses as the x for this identity), 2) $a - b = a + -b$, and 3) $a(b+c+d) = ab + ac + ad$ (on the second set of parentheses only) to replace the original expression with an equivalent expression that has a different form.

$$(x^2+2x-1)-(3x^2-4x+7)$$

3. Use the identities 1) $\sqrt{ab} = \sqrt{a}\sqrt{b}$ and 2) $\sqrt{x^2} = x$ $(whenever\ x \geq 0)$ (in that order) to replace the original expression with an equivalent expression that has a different form.

$$\sqrt{xy^2}$$

4. Use the identities 1) $ab = ba$ to reverse the order of the sub-expression x^2y at the top of the fraction, 2) $\frac{ab}{cd} = \frac{a}{c} \cdot \frac{b}{d}$ (whenever $c, d \neq 0$) to separate the x^2z from the other numbers and variables in both the top and bottom, 3) $\frac{x}{x} = 1$ (whenever $x \neq 0$) and 4) $x \cdot 1 = x$ (in that order) to replace the original expression with an equivalent expression that has a different form.

$$\frac{2x^2yz}{3x^2z} \qquad (x, z \neq 0)$$